Chapter (2): Beam Analysis

2.1 Loading on Beams:

2.2 Loading Types:

The loading on beam can be categorized to (Figure 2-1):

- Concentrated Load
  - Concentrated Force
  - Concentrated Moment
- Distributed Load
  - Uniformly Distributed Load (UDL)
  - Linearly Varying Distributed Load (LVDU)

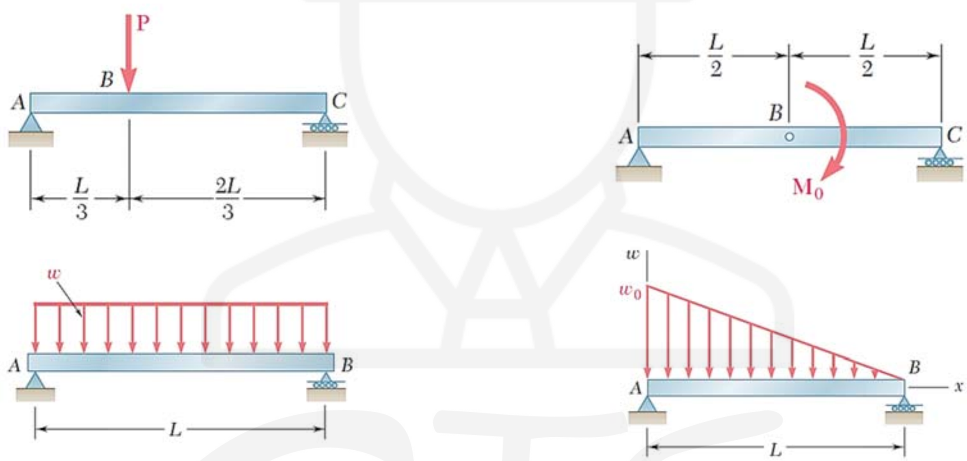


Figure 2-1: Loading types on beams

2.3 Support Types:

Supports on beams transfer the loads to the following structural member (usually a column) Three major types (Figure 2-2):

- Roller → Vertical reaction only
- Hinge → Vertical and horizontal reaction
- Fixed → Vertical and horizontal reaction + a bending moment

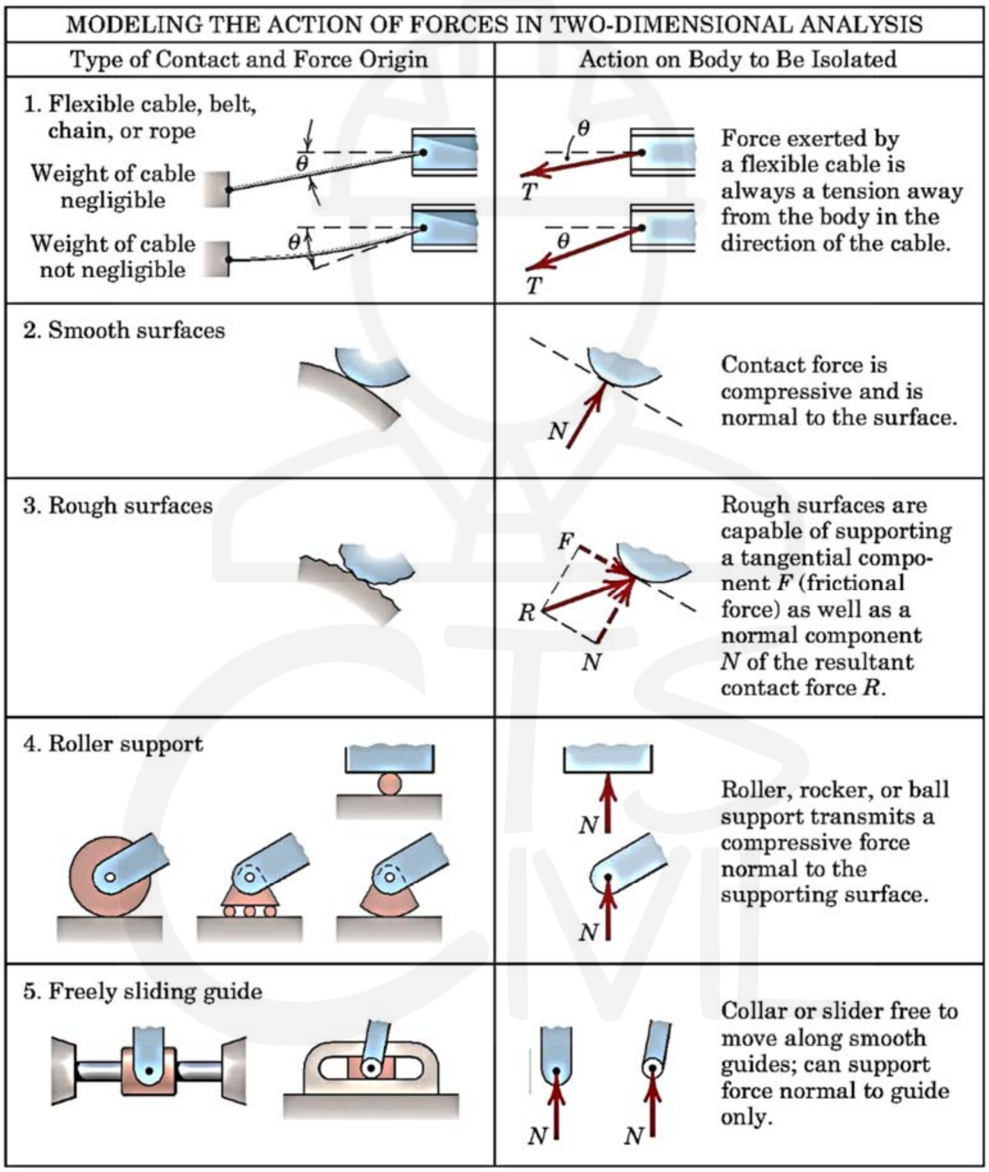
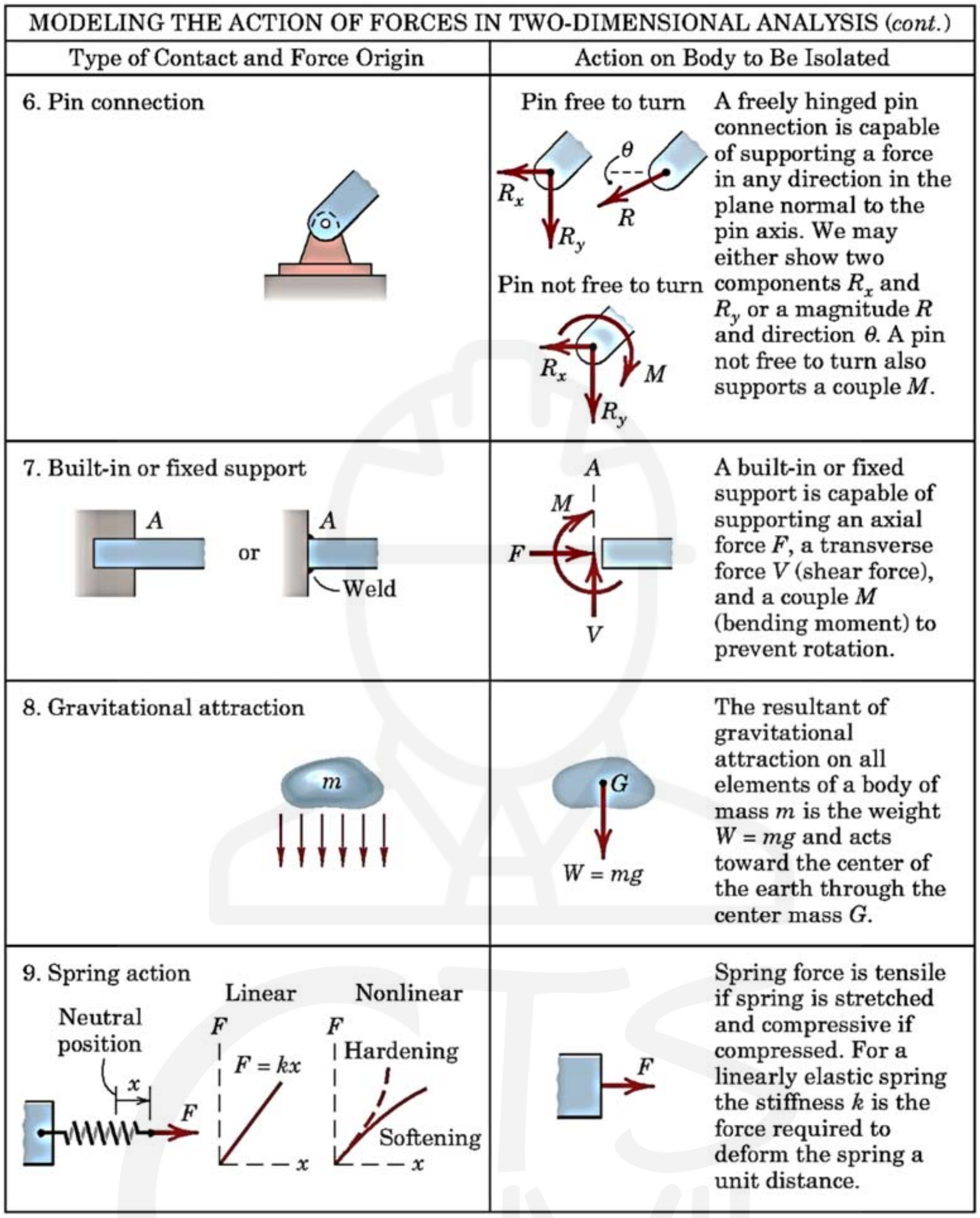


Figure 2-2: Beam reaction types



MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
6. Pin connection	A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support	A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction	The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action	Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

Figure 2-3: Beam reaction types (Continued)

2.4 Beam Types:

Beams can be divided into (Figure 2-4):

- **Statically determinate beams:**
  - Simply supported beams
  - One-sided over-hanging beam
  - Two-sided over-hanging beam
  - Cantilever beam
- **Statically indeterminate beams:**
  - Continuous beam
  - End-supported cantilever
  - Fixed at both ends

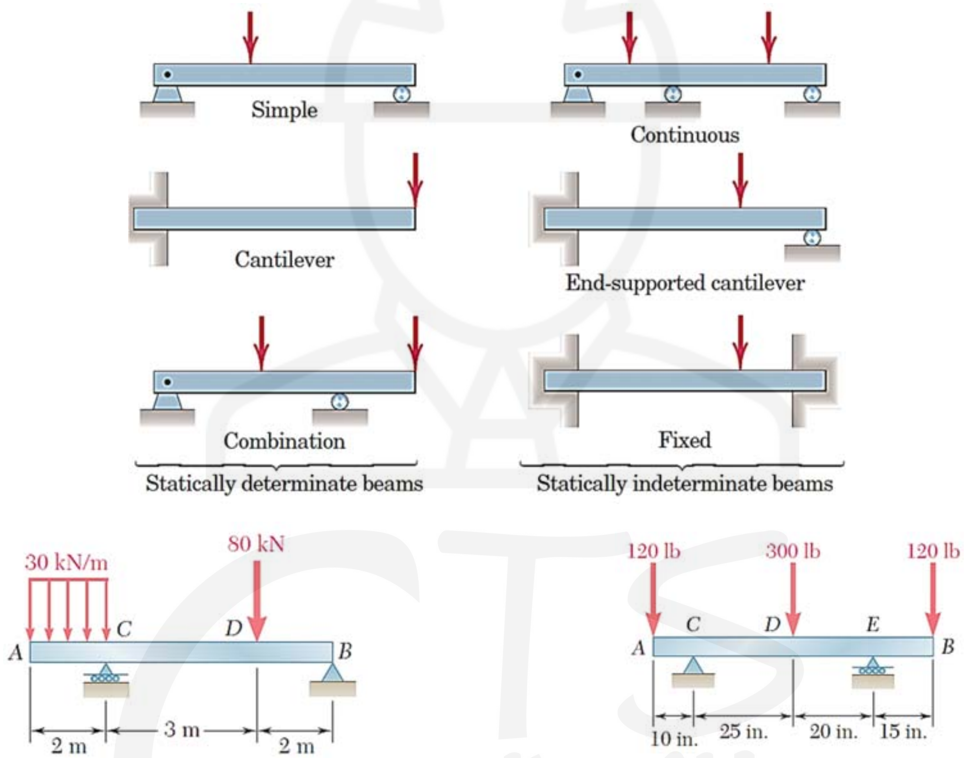


Figure 2-4: Beam types

2.5 Beam Reactions:

- Reactions on beams are developed due to the applications of the various loads on the beam.
- The reactions can be calculated (determinate beams only) by applying the three equations of equilibrium after drawing the free body diagram of the beam.
- The three equations of equilibrium are:

$$\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum M &= 0 \end{aligned} \qquad (2\text{-}1)$$

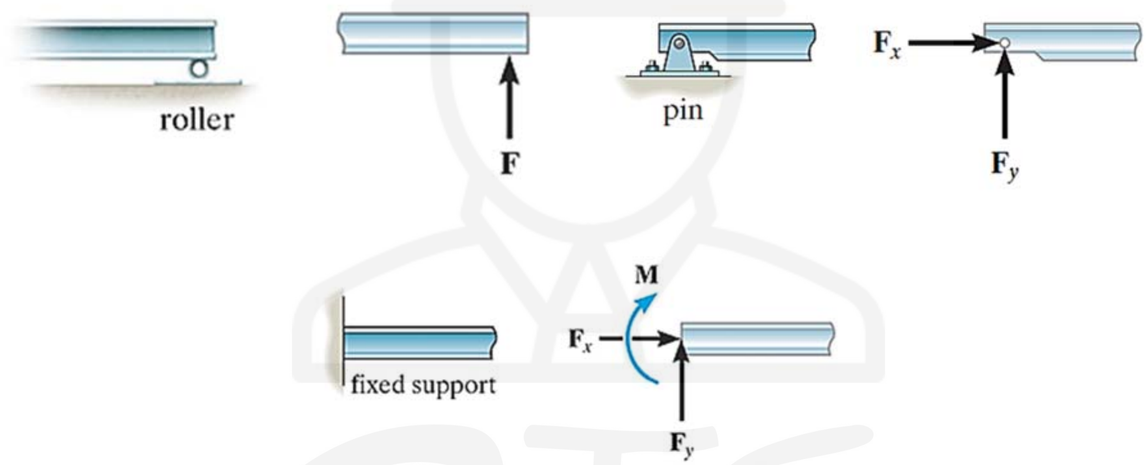


Figure 2-5: Beam reaction types

2.6 Sign Convention:

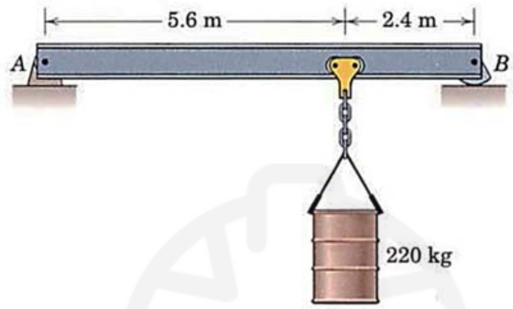
The positive sign convention used throughout the course is summarized in Figure 2-6. The positive x-direction is taken to the right, the positive y-direction is taken upward, and the positive moment is taken in the counter-clockwise direction.

Figure 2-6:The positive sign convention for forces and moment

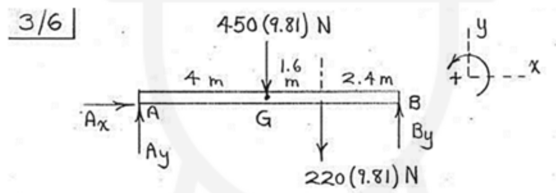
2.7 Examples:

Example (1):

The 450-kg uniform I-beam supports the load shown. Determine the reactions at the supports.



Solution:



From $\Sigma F_x = 0$, $A_x = 0$

$\Sigma M_A = 0: -450(9.81)4 - 220(9.81)(5.6) + B_y(8) = 0$, $\underline{B_y = 3720 \text{ N}}$

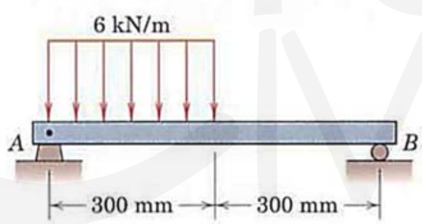
$\Sigma F_y = 0: A_y - 450(9.81) - 220(9.81) + 3720 = 0$

$\underline{A_y = 2850 \text{ N}}$

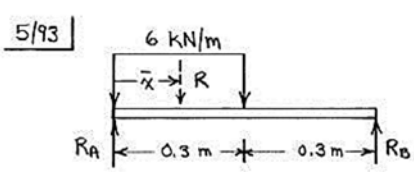
Example (2):

Determine the reactions at A and B for the beam subjected to the uniform load distribution.

Ans. $R_A = 1.35$ kN, $R_B = 0.45$ kN



Solution:



$R = 6(0.3) = 1.8 \text{ kN}$ @ $\bar{x} = \frac{1}{2}(0.3) = 0.15 \text{m}$

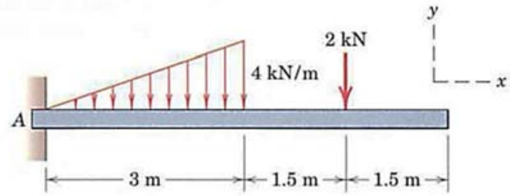
$\circlearrowleft + \Sigma M_A = 0: R_B(0.6) - 1.8(0.15) = 0$, $\underline{R_B = 0.45 \text{ kN}}$

$+\uparrow \Sigma F = 0: 0.45 - 1.8 + R_A = 0$, $\underline{R_A = 1.35 \text{ kN}}$

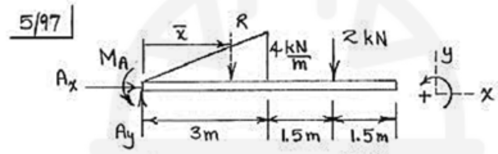
Example (3):

5/97 Determine the reactions at A for the cantilever beam subjected to the distributed and concentrated loads.

Ans. $A_x = 0$, $A_y = 8$ kN, $M_A = 21$ kN·m



Solution:



$R = \frac{1}{2}(3)(4) = 6$ kN @ $\bar{x} = \frac{2}{3}(3) = 2$m

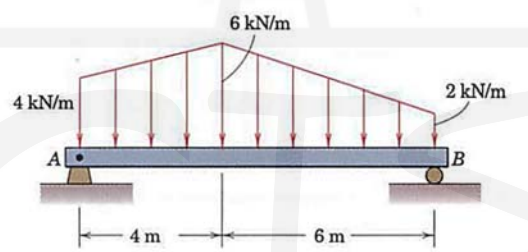
$\Sigma M_A = 0: \; M_A - 6(2) - 2(4.5) = 0, \quad \underline{M_A = 21 \text{ kN·m}}$

$\Sigma F_y = 0: \; A_y - 6 - 2 = 0, \quad \underline{A_y = 8 \text{ kN}}$

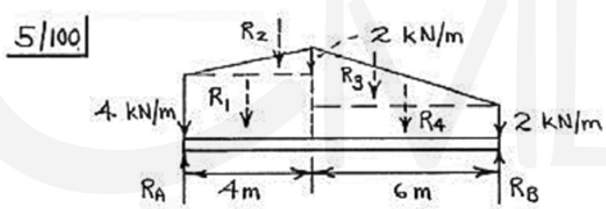
$\Sigma F_x = 0: \; \underline{A_x = 0}$

Example (4):

5/100 Calculate the support reactions at A and B for the beam subjected to the two linearly varying load distributions.



Solution:



$R_1 = 4(4) = 16$ kN, $\quad R_2 = \frac{1}{2}(2)(4) = 4$ kN

$R_3 = \frac{1}{2}(4)(6) = 12$ kN, $\quad R_4 = 2(6) = 12$ kN

$\circlearrowleft \Sigma M_A = 0: \; 16(2) + 4(\frac{2}{3}4) + 12(4 + \frac{1}{3}6)$

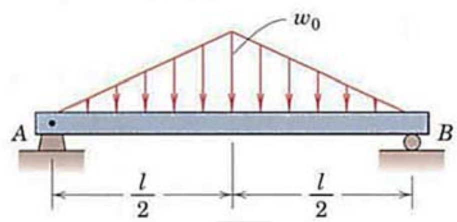
$+ 12(4+3) - 10 R_B = 0, \quad \underline{R_B = 19.87 \text{ kN}}$

$+\uparrow \Sigma F = 0: \; R_A + 19.87 - (16 + 4 + 12 + 12) = 0$

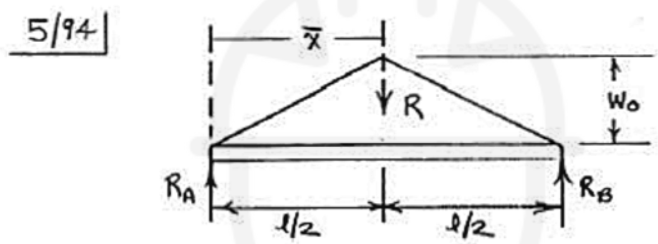
$\underline{R_A = 24.1 \text{ kN}}$

Example (5):

5/94 Determine the reactions at the supports A and B for the beam loaded as shown.



Solution:



$$R = 2\,\tfrac{1}{2}(w_0)\left(\tfrac{l}{2}\right) = \tfrac{1}{2} w_0 l \quad @\ \bar{x} = \tfrac{l}{2}$$

$$\circlearrowleft + \sum M_A = 0: \quad R_B(l) - \tfrac{1}{2} w_0 l \left(\tfrac{l}{2}\right) = 0, \quad \underline{R_B = \tfrac{1}{4} w_0 l}$$

$$+\uparrow \sum F = 0: \quad \tfrac{1}{4} w_0 l - \tfrac{1}{2} w_0 l + R_A = 0, \quad \underline{R_A = \tfrac{1}{4} w_0 l}$$

2.8 Internal Forces in Beams:

Internal forces were defined as the forces and couples exerted on a portion of the structure by the rest of the structure.

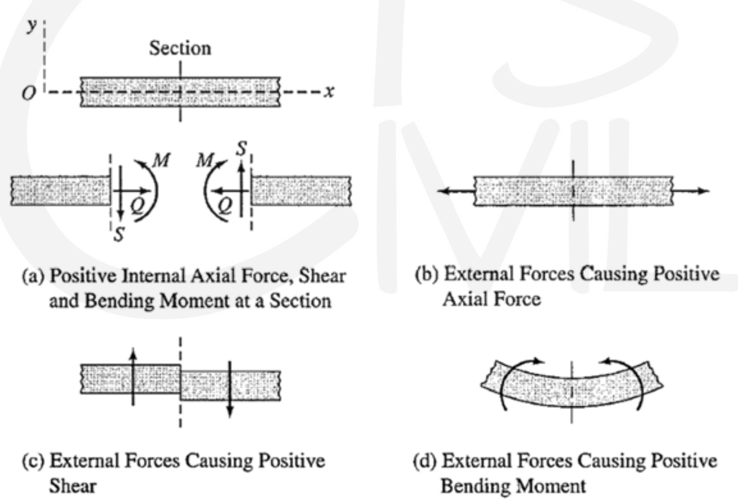


Figure 2-7: Sign convention for axial force, shear force, and bending moment